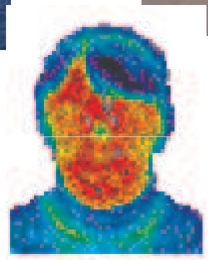
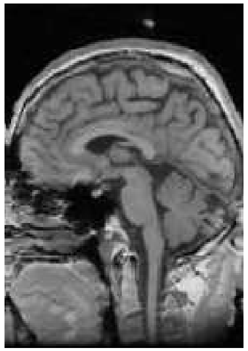
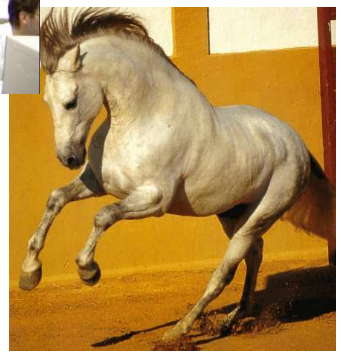
Images

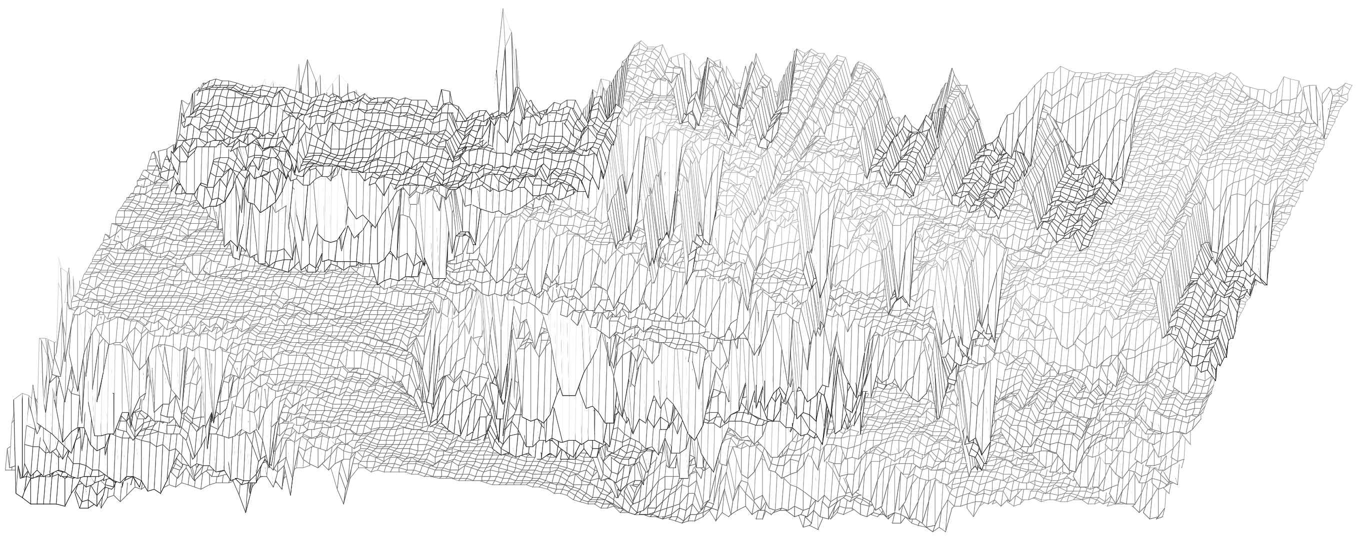
images: what are they? how are they obtained?





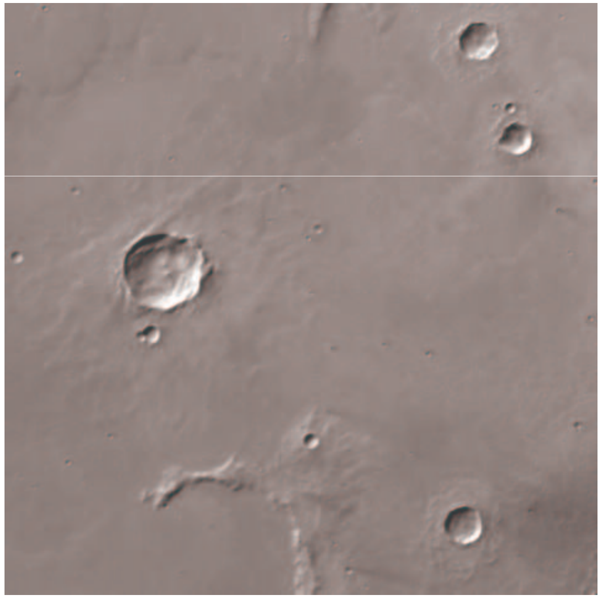


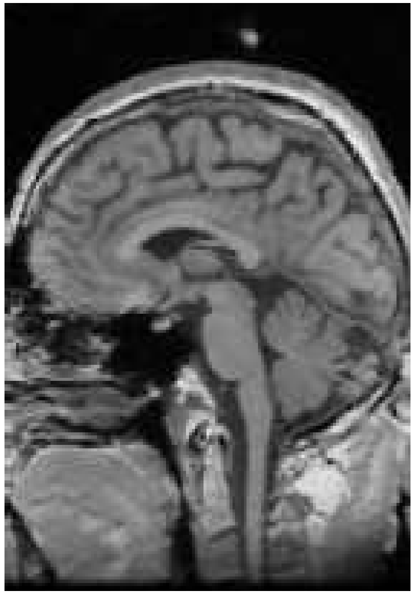
what is shown in this image



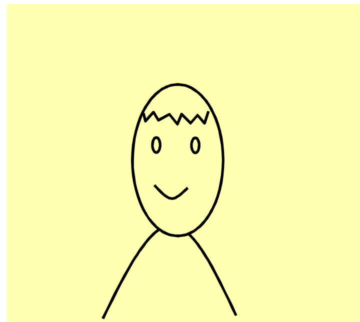
what is na image

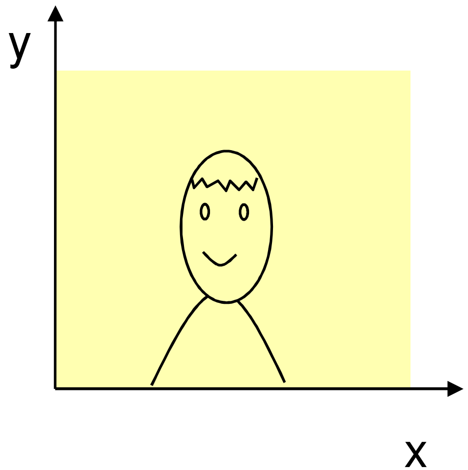
images describe the evolution of physical variables (intensity, color, reflectance, condutivity) in a plane or in a 3D volume.

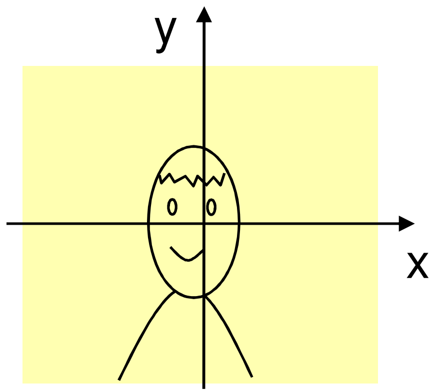


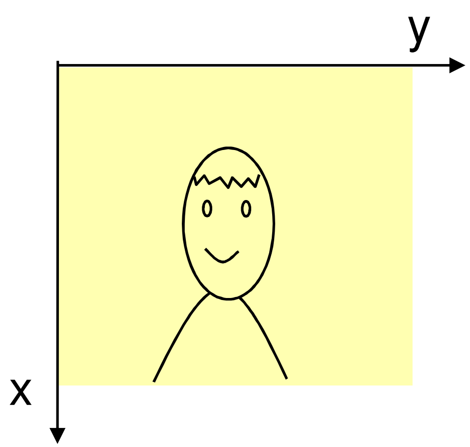


coordinate systems





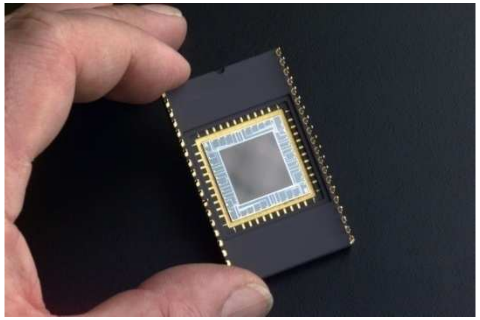




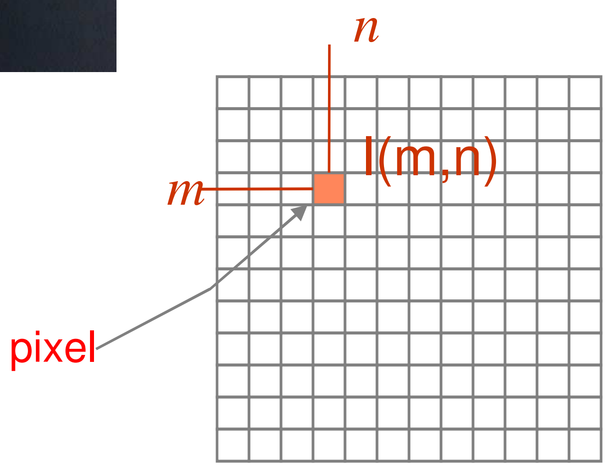
most common choice

discrete images

Most electronic sensors produce discrete images defined on a grid of points



sCCD sensor



42	45	49	57	65	54	66	63	42	64	95
73	72	68	67	69	75	59	62	55	35	37
82	83	83	84	85	81	79	90	77	76	73
96	95	95	$\boxed{92}$	90	93	88	79	95	88	93
93	91	93	95	95	96	97	104	94	96	104
110	107	102	104	110	107	103	102	110	97	106
109	112	113	109	105	107	101	108	116	115	111
109	107	109	112	113	101	119	128	143	121	122
114	114	115	115	113	119	146	154	115	110	116
125	125	120	118	121	160	140	112	118	116	122
114	110	115	137	161	132	119	119	107	124	120

pixel – **pic**ture **el**ement

each pixel is characterized by its position and value

Matriz notation

A discrete image is a matrix.

Na image I(x) woth M lines and N columns can be represented by a matrix of dimension MxN or by a vector of dimension MNx1.

Exemplo

42	45	49	57
73	72	68	67
82	83	83	84
96	95	95	92
93	91	93	95

$$I = \begin{bmatrix} 42 & 45 & 49 & 57 \\ 73 & 72 & 68 & 67 \\ 82 & 83 & 83 & 84 \\ 96 & 95 & 95 & 92 \\ 93 & 91 & 93 & 95 \end{bmatrix}$$

$$i = \begin{bmatrix} 42 \\ 73 \\ 82 \\ 96 \\ \vdots \\ \vdots \\ 92 \\ 95 \end{bmatrix}$$

p.ex., column concatenation

vector space of images

sum

The sum of two discrete images u,v with M lines and N columns is a discrete image w such that

$$w(x)=u(x)+v(x)$$

multiplication by a scalar

The multiplication of na image u with M linhes and N columnsby a constant α is a discrete image w, such that

$$w(x)=\alpha\, u(x)$$

The set of images with M lines and N columns is a vector space.

mixing two images

$$w(x)= \alpha\, u(x) + \beta\, v(x)$$

2 images are not enough to synthesize a wide variety of images.

How many do we need?   (discussion)

exercises

1)

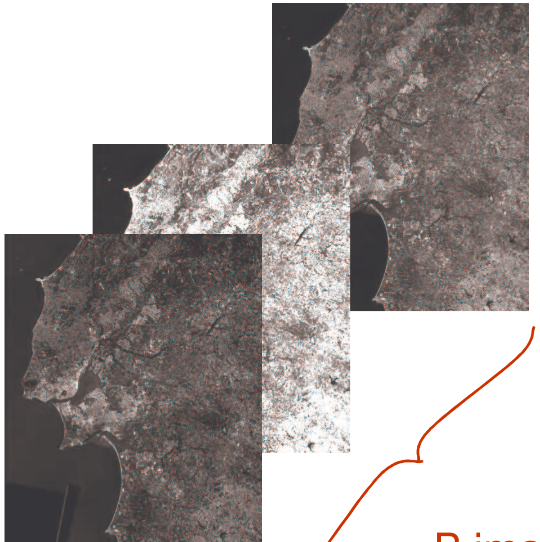
$$U = \begin{bmatrix} 1 & 2 \\ -1 & 0 \\ 2 & -1 \end{bmatrix} \quad V = \begin{bmatrix} -1 & 0 \\ 0 & 3 \\ -1 & 2 \end{bmatrix} \quad U + \mathrm{W} = \begin{bmatrix} 0 & 2 \\ -1 & 3 \\ 1 & 1 \end{bmatrix} \quad 3\mathrm{U} = \begin{bmatrix} 3 & 6 \\ -3 & 0 \\ 6 & -3 \end{bmatrix}$$

2) Define a base for the set of 3x2 discrete images.

multi-dimensional images

A satelite image consists of B real images obtained at different frequency bands.

The value of the image I at a position (m,n) is a vector



B imagens

$$I(m,n) = \begin{bmatrix} I(m,n,0) \\ I(m,n,1) \\ \vdots \\ \vdots \\ I(m,n,B-1) \end{bmatrix}$$

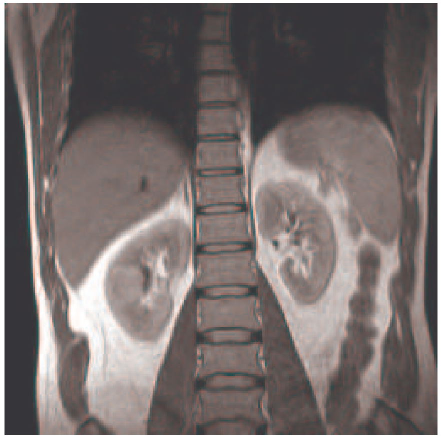
Another example is the color images.

m,n - spatial coordinates

B - number of frequency bands

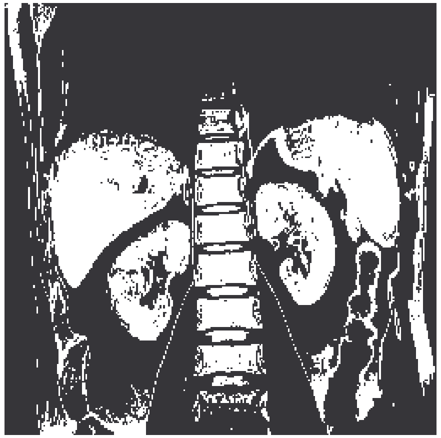
binary images

I(m,n)



MRI dos rins

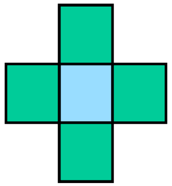
B(m,n)



Mozart

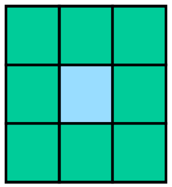
topological concepts

what is the neighborhood of a pixel *(i,j)* ?



neighborhood 4

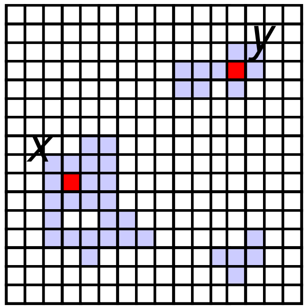
$$V_4(i,j) = \{(i-1,j),(i+1,j),(i,j-1),(i,j+1)\}$$



neighborhood 8

$$V_8(i,j) = \{(i-1,j-1),(i-1,j),(i-1,j+1),(i,j-1),\\ (i,j+1),(i+1,j-1),(i+1,j),(i+1,j+1)\}$$

conectivity



binary image

are x, y connected?

NO

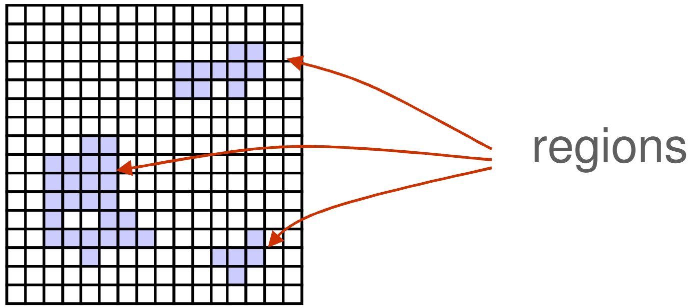
conectivity: two pixels x, y of a binary image are connected iif (if and only if) there a sequence of points $z^1, z^2, \ldots, z^n$:

$$z^1 = x, z^n = y$$

$$z^i, z^{i+1} \quad \text{are neighbors} \quad \forall i \in \{1, 2, \ldots, n-1\}$$

$$u(z^i) = 1 \quad \forall \mathrm{i} = 1, \ldots, \mathrm{n}$$

regions



Region R is a set of fully connected pixels i.e.,

$$\forall x, y \in R, \ \text{x, y are connected}$$

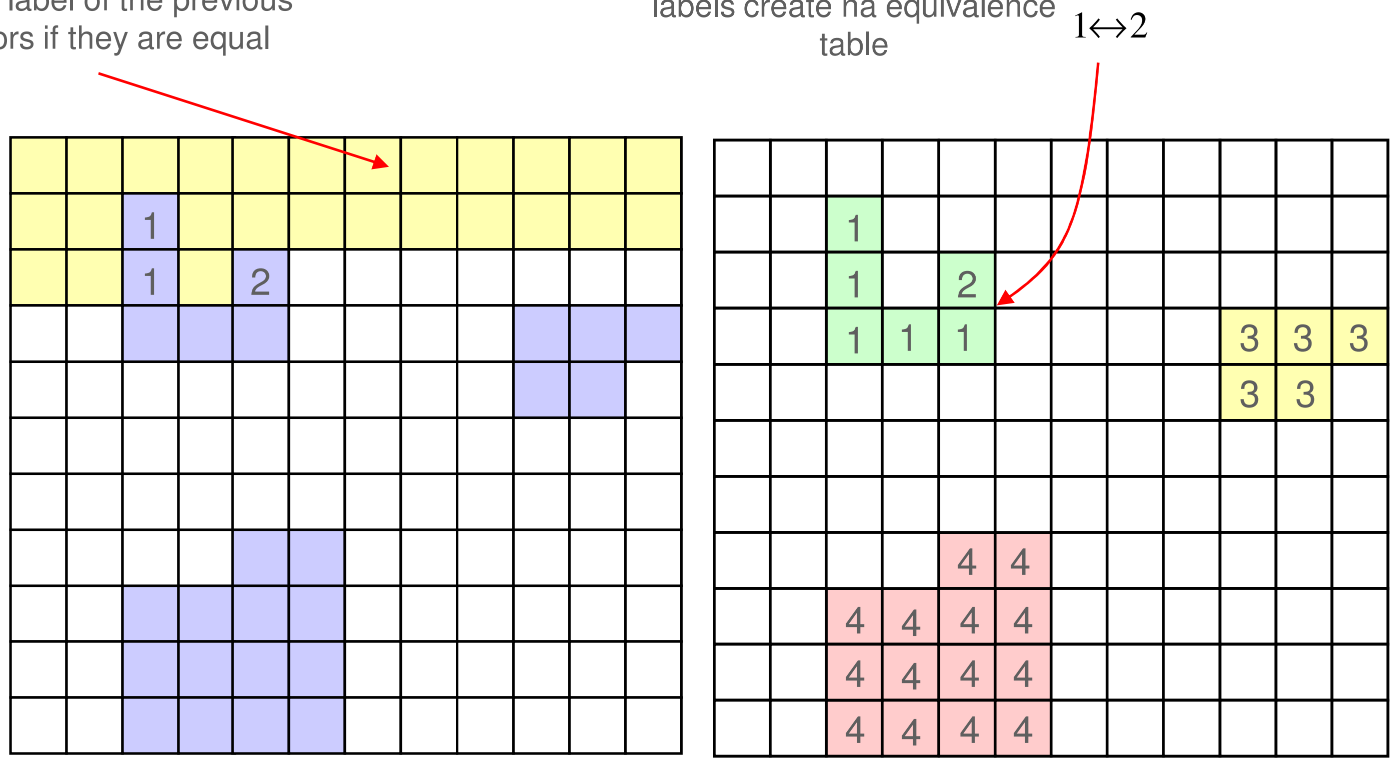
A maximal set of connected pixels is called a connected component.

Algorithm for connected components

The algorithm assigns different labels to pixels belonging to different connected components

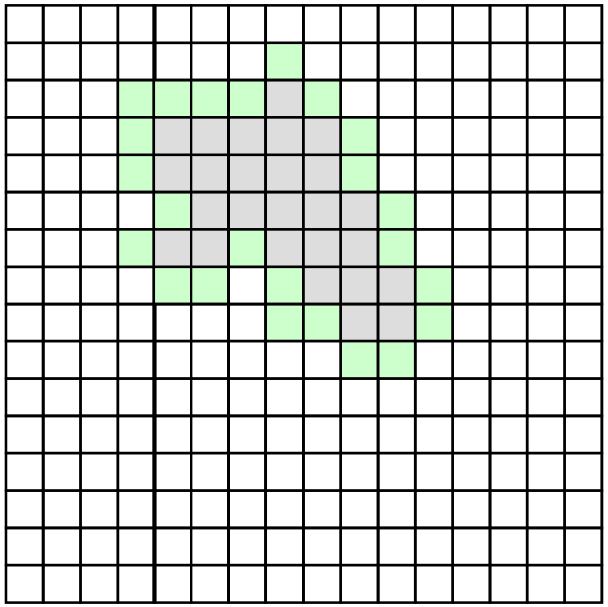
ssign the label of the previous neighbors if they are equal

If the neighbors have different labels create na equivalence table



Boundary

The boundary of a region R is the set of R pixels whose neighborhood is not contained in R.



summary: image types

Image $I : D \quad C$

Domain D

R^2 or interval $[a,b]x[c,d] \subset R^2$ — continuous image

Z^2 or interval $\{0,\ldots,M-1\}x\{0,\ldots,N-1\} \subset Z^2$ — discrete image

Codomain C

R	monocromatic image	ex: intensity image
R^n	multidimensional image	ex: color image
$\{0,1\}$	binary image	ex: binary segmentation
$\{0, \ldots, L-1\}$	label image	ex: segmentação multi-objectos

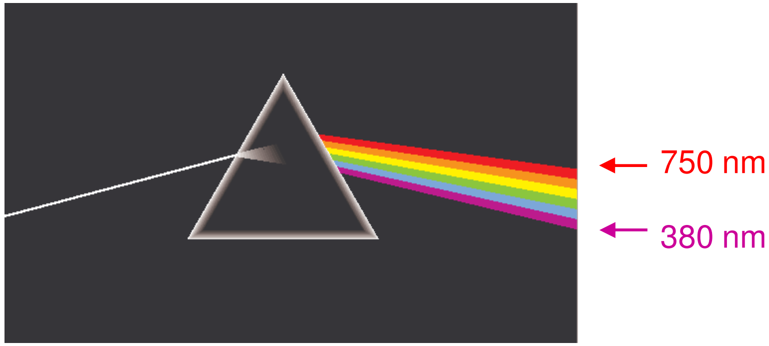
cor

desafio

www.rtribatejo.org

o que é a cor ? como é que se indexa uma imagem através da cor ?

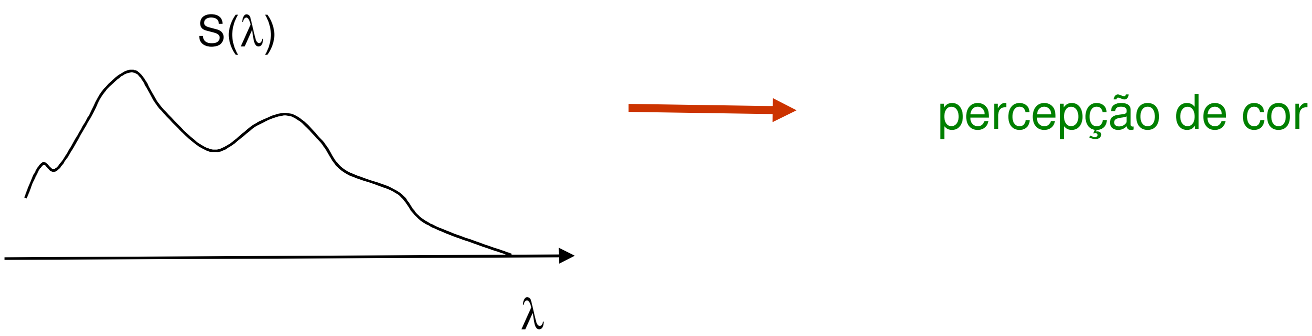
espectro visível



o que é a cor?

A cor é uma percepção humana.

É a resposta dos sensores localizados na retina ao espectro electromagnético na gama 380-750 nm.



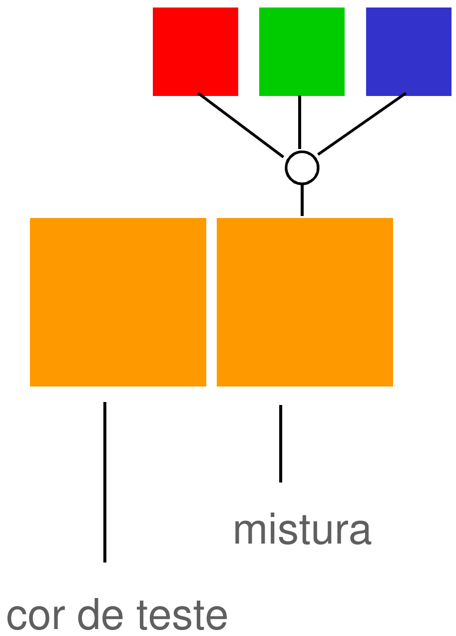
é preciso uma descriçao detalhada do espectro para caracterizar a cor? Não!

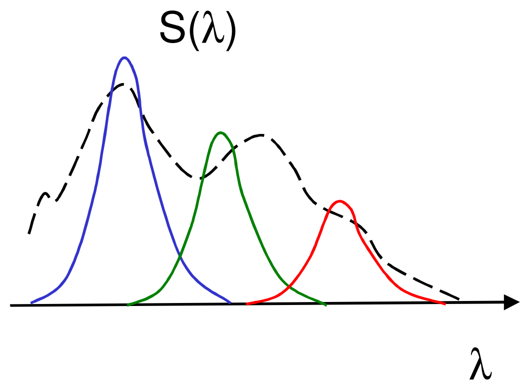
teoria tricromática

Desenvolvida por Tomas Young (sec. XVIII) Helmoltz (sec. XIX)

Helmoltz mostrou que qualquer (?) cor se pode sintetizar através da mistura de três cores primárias fixas

experiência





$$\hat{S}(\lambda) = c_1 P_1(\lambda) + c_2 P_2(\lambda) + c_3 P_3(\lambda)$$

pouco intuitivo, com 3 cores primárias não se pode aproximar bem o espectro S(λ)!

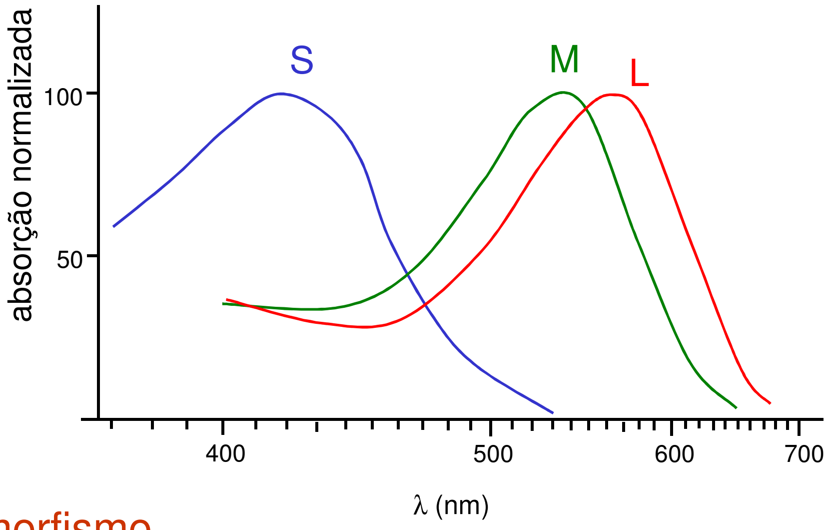
Young acreditava que havia 3 estruturas sensíveis à cor na retina.

verdade?...

retina

Tal como Young previu, há três tipos de células foto sensíveis na retina designados por cones.

Os três tipos de cones têm curvas de sensibilidade espectral (absorção) diferentes.



Dois espectros podem ser diferentes e causar a mesma percepção de cor. meromorfismo

Resposta do cone i a um espectro S(λ)

$$R_i(S) = \int S(\lambda) S_i(\lambda) d\lambda$$

A percepção de cor é idêntica se as respostas dos cones forem iguais.

ajuste de cor

A mistura de cores primárias tem a mesma cor que o espectro incidente se

$$R_i(S) = R_i(\hat{S}), \ \mathrm{i} = 1,2,3$$

$$\int \ S(\lambda)S_i(\lambda)d\lambda = \int \ \sum_{k=1}^{3} c_k P_k(\lambda)S_i(\lambda)d\lambda \quad i = 1,2,3$$

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix}$$

$$R_{ij} = \int \ P_j(\lambda)S_i(\lambda)d\lambda$$

$$r_i = \int \ S(\lambda)S_i(\lambda)d\lambda$$

a projecção S(λ) c é linear e fica completamente caracterizada se soubermos como se transforma cada risca espectral $\delta(\lambda - \lambda_0)$.

funções de ajuste espectral

As cores monocromáticas $\delta(\lambda - \lambda_0)$ podem ser sintetizadas através da mistura de três cores primárias.

$$M(\lambda) = f_1(\lambda_0)P_1(\lambda) + f_2(\lambda_0)P_2(\lambda) + f_3(\lambda_0)P_3(\lambda)$$

As funções $f_i(\lambda)$ designam-se por funções de ajuste espectral.

Os pesos da mistura M, podem ser obtidos a partir das funções de ajuste espectral.

$$c_i = \int f_i(\lambda)\, S(\lambda)\, d\lambda$$

S – espectro da radiação luminosa

CIE 1931

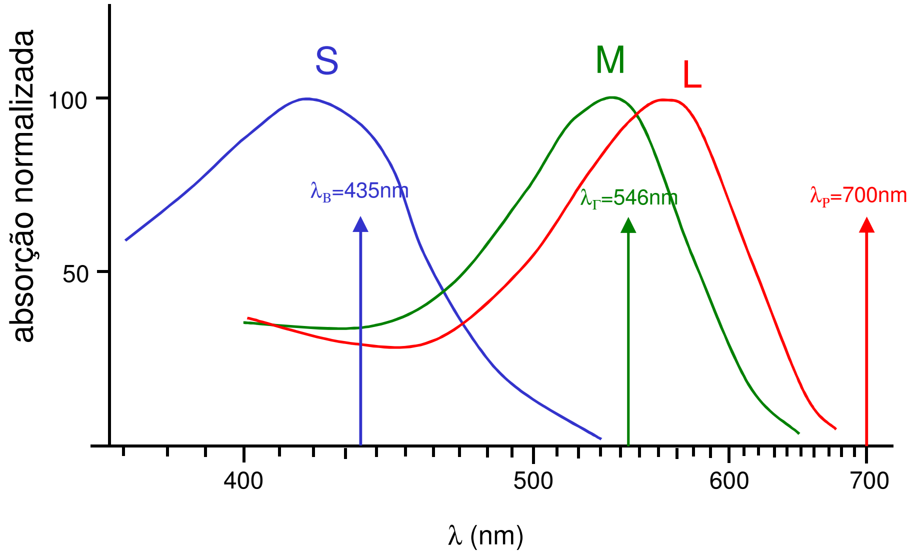
Guild and Wright fizeram experiências de ajuste de cores monocromáticas usando 3 cores primárias R,G,B monocromáticas.

Cores primárias monocromáticas

$$\lambda_1 = 700 nm \text{ (red)}$$

$$\lambda_2 = 546.1 nm \text{ (green)}$$

$$\lambda_3 = 435.8 nm \text{ (blue)}$$



Que quantidade de cada cor primária é necessário misturar para gerar a cor dum espectro monocromático com comprimento de onda λ_0 funções de ajuste espectral

Este estudo esteve na base da norma CIE de 1931

Espaço RGB

CIE, 1931 – sistema espectral primário

$$P_j(\lambda) = \delta(\lambda - \lambda_i)$$

$\lambda_1 = 700nm$ (R) $\lambda_2 = 546.1nm$ (G) $\lambda_3 = 435.8nm$ (B)

funções de ajuste espectral

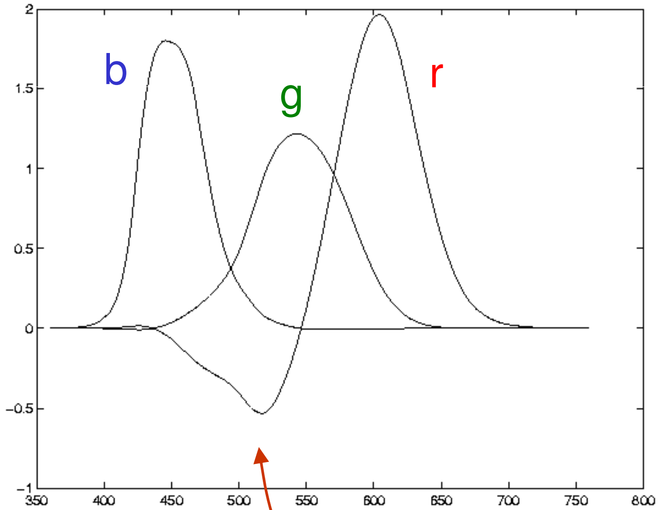
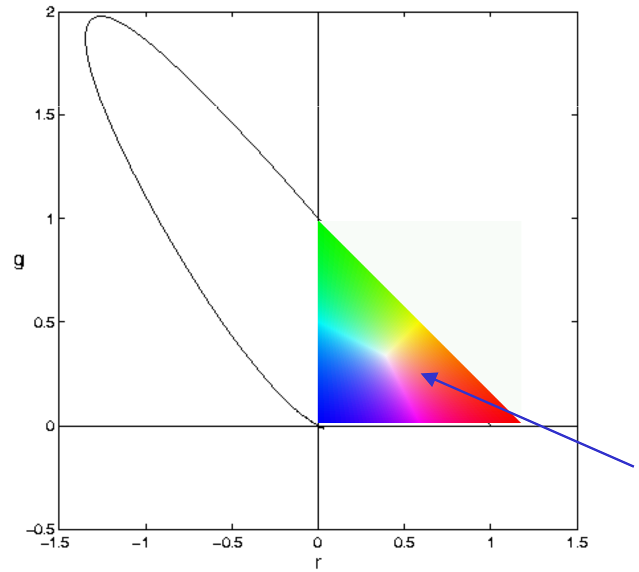


diagrama de cromaticidade



$$r = \frac{R}{R+G+B}$$

$$g = \frac{G}{R+G+B}$$

cores realizáveis

os coeficientes da mistura podem ser negativos?

espaço XYZ

CIE

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.490 & 0.310 & 0.200 \\ 0.177 & 0.813 & 0.011 \\ 0.000 & 0.010 & 0.990 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

Y - luminância

funções de ajuste espectral

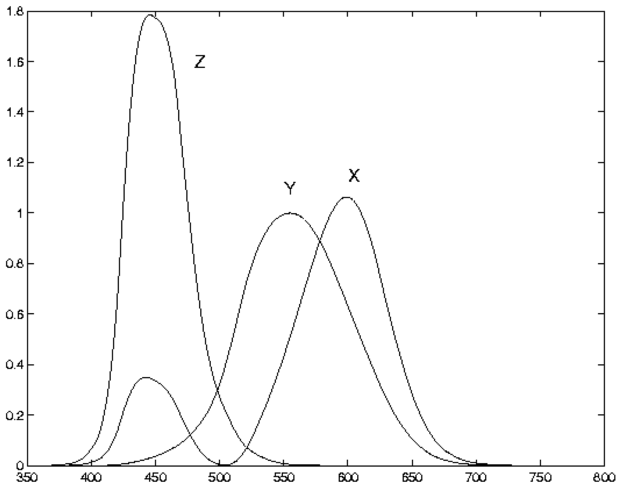
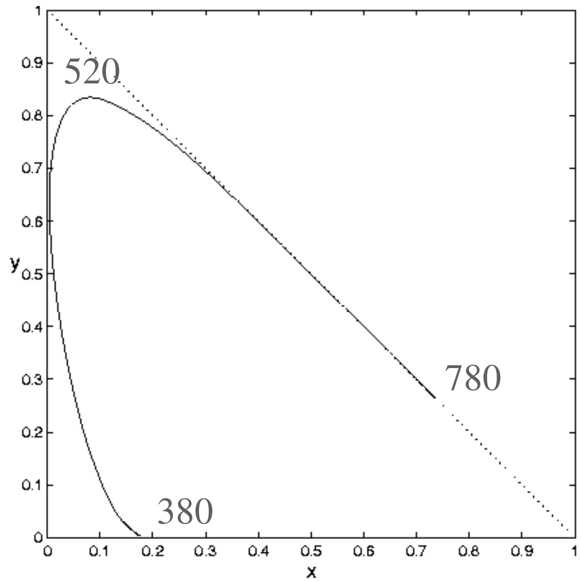


diagrama de cromaticidade

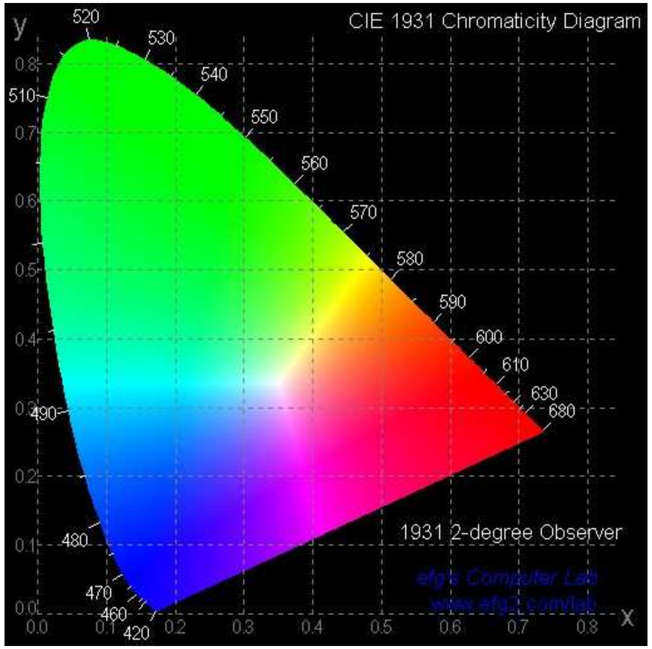


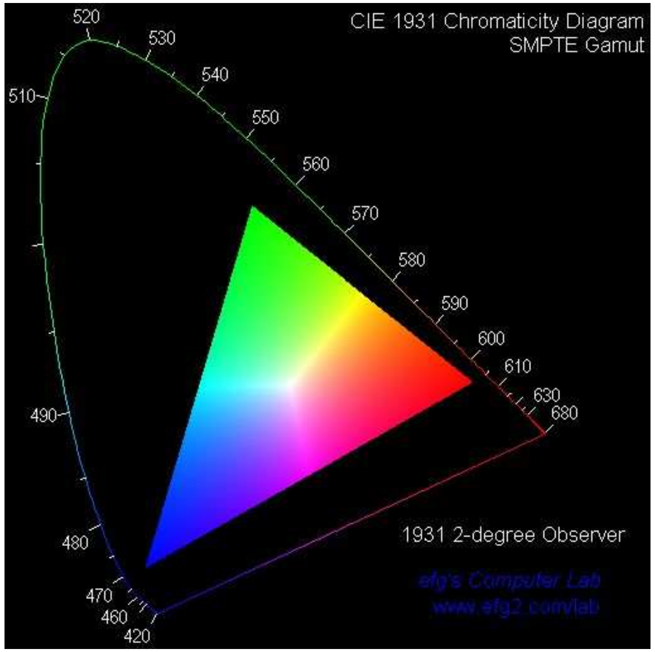
$$x = \frac{X}{X+Y+Z}$$

$$y = \frac{Y}{X+Y+Z}$$

Os coeficientes são sempre positivos mas as cores primárias são irrealizáveis !

diagrama de cromaticidade CIE 1931





Componentes de cor

R

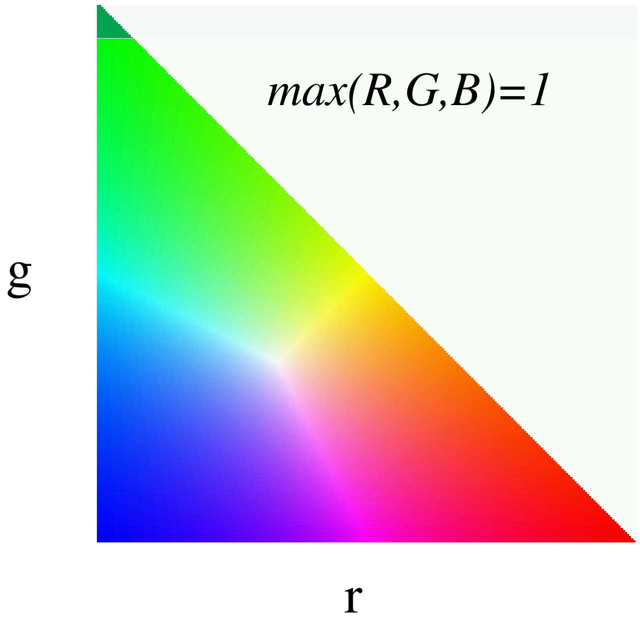
G

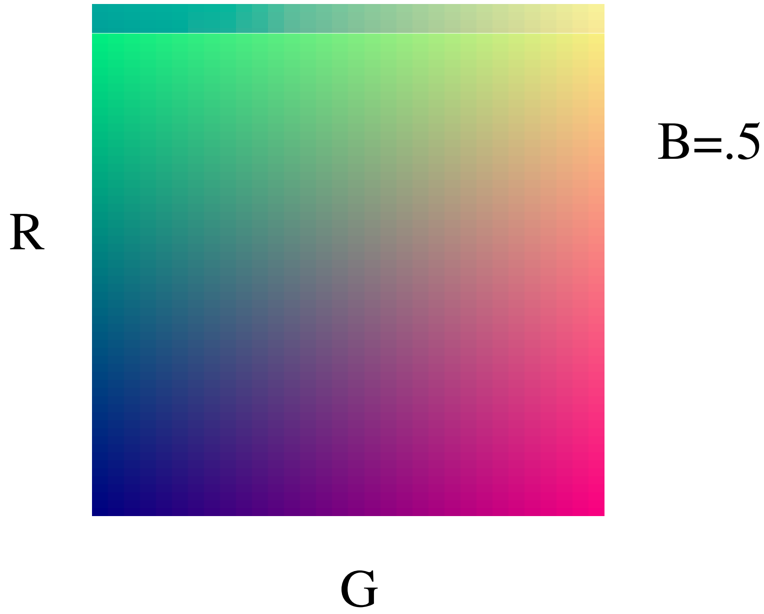
B

Colorimetria

Como medir diferenças entre cores ?

Os espaços anteriores não são uniformes do ponto de vista perceptual: a diferença perceptual entre duas cores não é proporcional à distância euclidiana entre as respectivas coordenadas de cor.



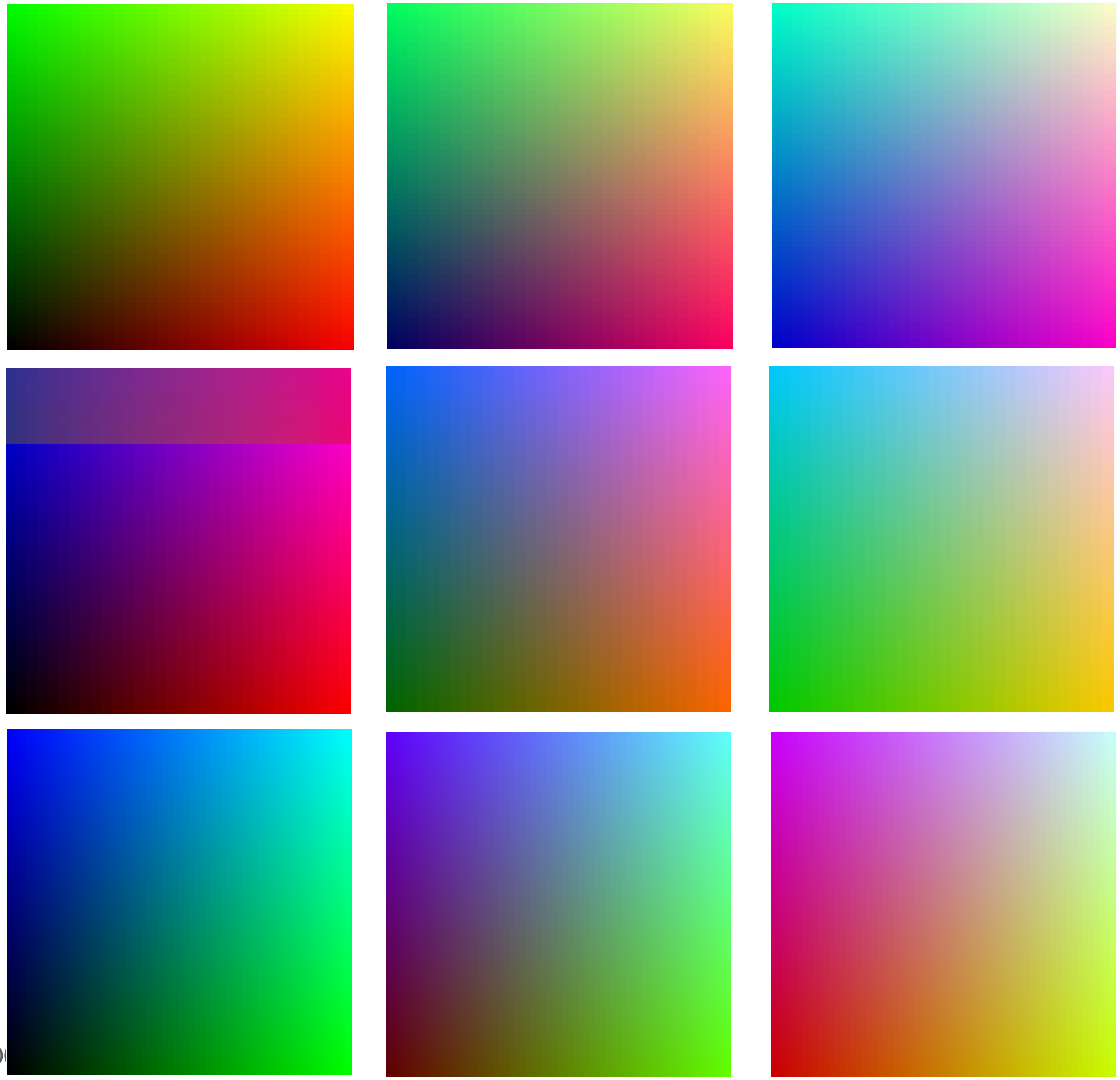


planos de cor

RG

RB

GB



B=0,100,200

G=0,100,200

R=0,100,200

Métricas de cor

espaços de Riemann

$$(ds)^2 = \sum_{i=1}^{3}\sum_{j=1}^{3} c_{ij} dX_i dX_j$$

os coeficientes c_{ij} variam ao longo do espaço

coordenadas não lineares

aplicar uma transformação não linear às coordenadas de cor mantendo a distância euclidiana.

Espaço LAB

CIE 1976 - coordenadas não lineares

$$L^* = 116\left(\frac{Y}{Y_0}\right)^{1/3} - 16$$

$$a^* = 500\left[\left(\frac{X}{X_0}\right)^{1/3} - \left(\frac{Y}{Y_0}\right)^{1/3}\right]$$

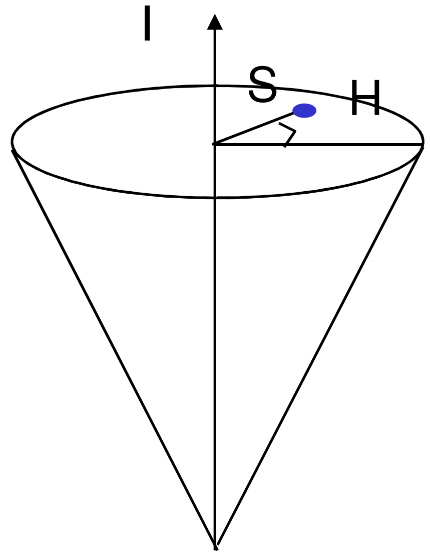
$$b^* = 200\left[\left(\frac{Y}{Y_0}\right)^{1/3} - \left(\frac{Z}{Z_0}\right)^{1/3}\right]$$

L* - brilho
a* - conteúdo vermelho-verde
b* - conteúdo amarelo-azul
X_0, $Y_{0,}$, Z_0 – coordenadas do branco de referência

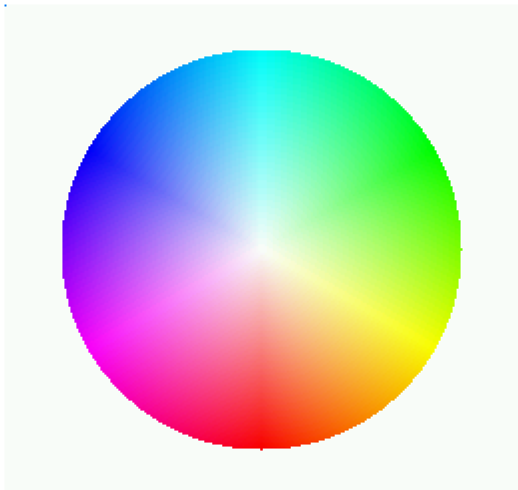
São apropriadas para colorimetria: medição de diferenças de cor

espaço HSI

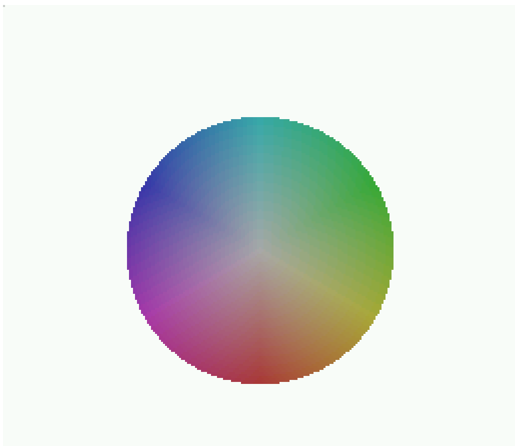
H – tom
S – saturação
I – intensidade



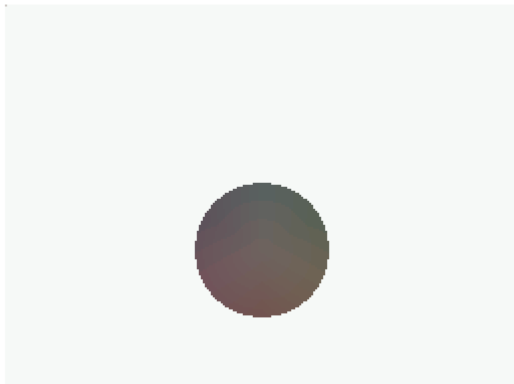
Representação próxima da descrição humana de cor



I=1



I=2/3



I=1/3

Fórmulas de conversão

Gonzalez & Woods

$$I = \frac{1}{3}(R + G + B)$$

$$S = 1 - \frac{3\min(R,G,B)}{R+G+B}$$

$$H = \begin{cases} \frac{1}{2\pi}\cos^{-1}\left(\frac{0.5\,(R-G+R-B)}{\sqrt{(R-G)^2+(R-B)(G-B)}}\right) & \text{B>G} \\ 1-\frac{1}{2\pi}\cos^{-1}\left(\frac{0.5\,(R-G+R-B)}{\sqrt{(R-G)^2+(R-B)(G-B)}}\right) & \text{B≤G} \end{cases}$$

$$B = \frac{1}{3}(1\text{-}S)\,,\quad R = \frac{1}{3}[1+\frac{S\cos\theta}{\cos(\theta-\frac{\pi}{3})}],\quad G = 1 - (B+R)\,,\qquad \theta = \cos(2\pi H),\qquad 0 \le H < \frac{1}{3}$$

$$R = \frac{1}{3}(1\text{-}S)\,,\quad G = \frac{1}{3}[1+\frac{S\cos\theta}{\cos(\theta-\frac{\pi}{3})}],\quad B = 1 - (R+G),\qquad \theta=\cos(2\pi(H-\frac{1}{3})),\qquad \frac{1}{3}\le H<\frac{2}{3}$$

$$G = \frac{1}{3}(1\text{-}S)\,,\quad B = \frac{1}{3}[1+\frac{S\cos\theta}{\cos(\theta-\frac{\pi}{3})}],\quad R = 1 - (G+B)\,,\quad \theta = \cos(2\pi(H-\frac{2}{3})),\qquad \frac{2}{3} \le H < 1$$

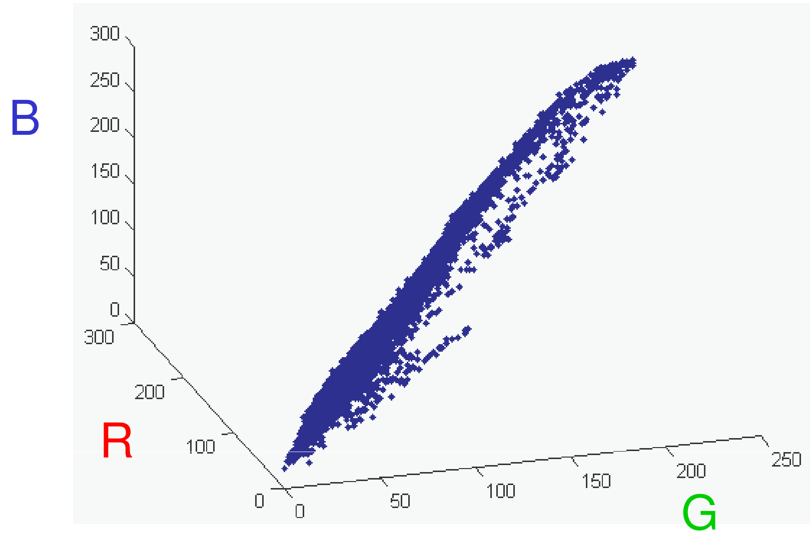
como descrever o conteúdo de cor de uma imagem ?

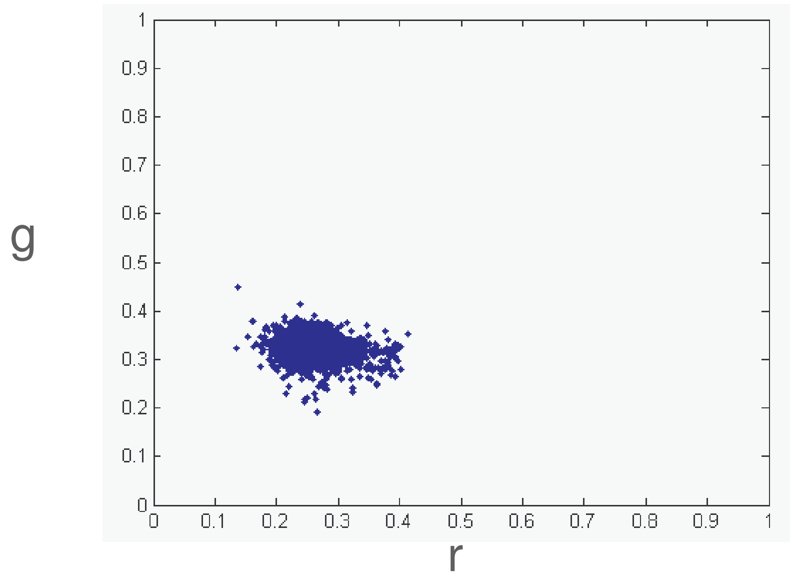
where is wally?

ou um problema mais realista

estudantes de piv bem dispostos!

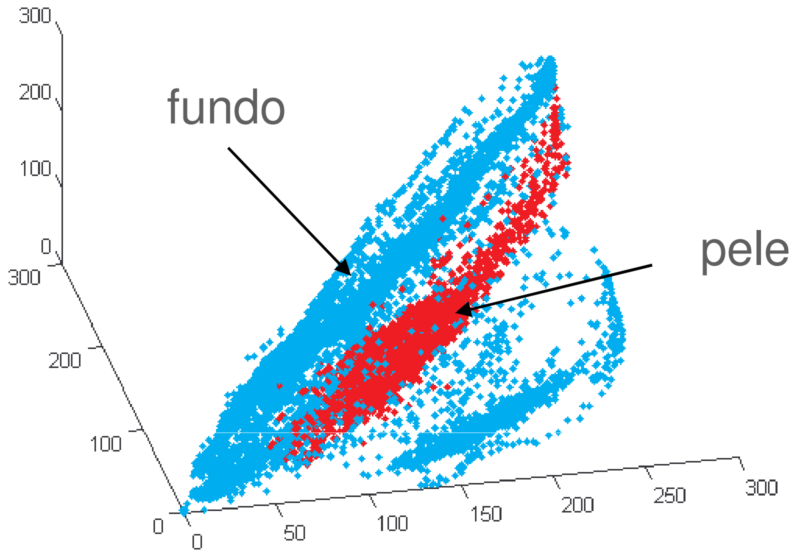
exemplo: camisola

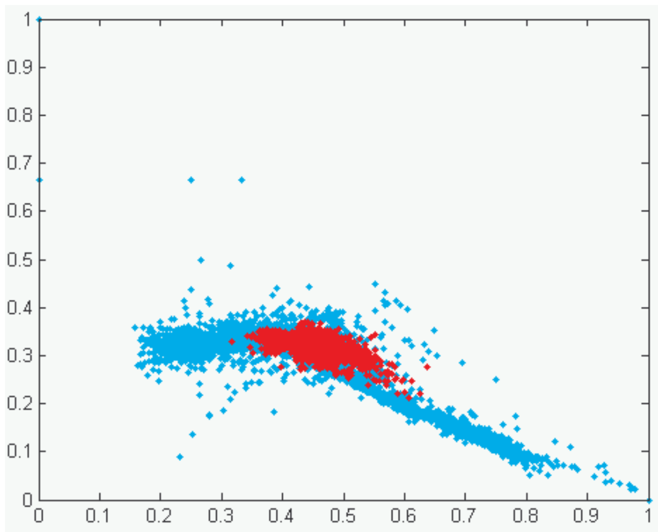




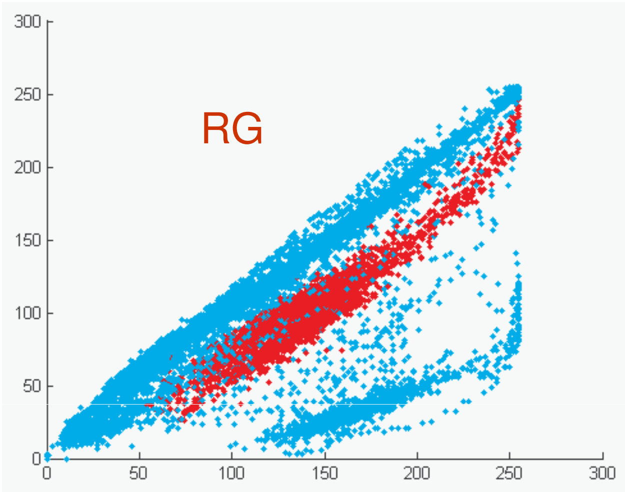
as variações são menores no espaço rg

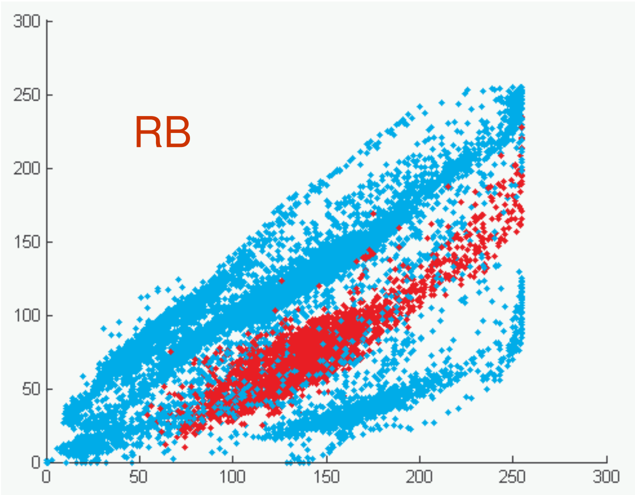
exemplo: pele

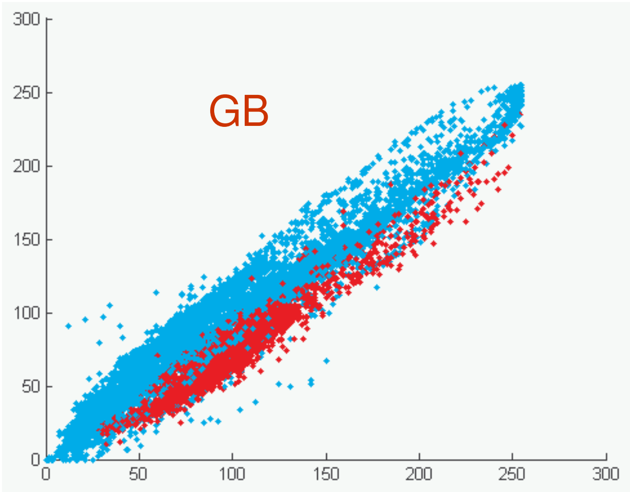




exemplo - pele

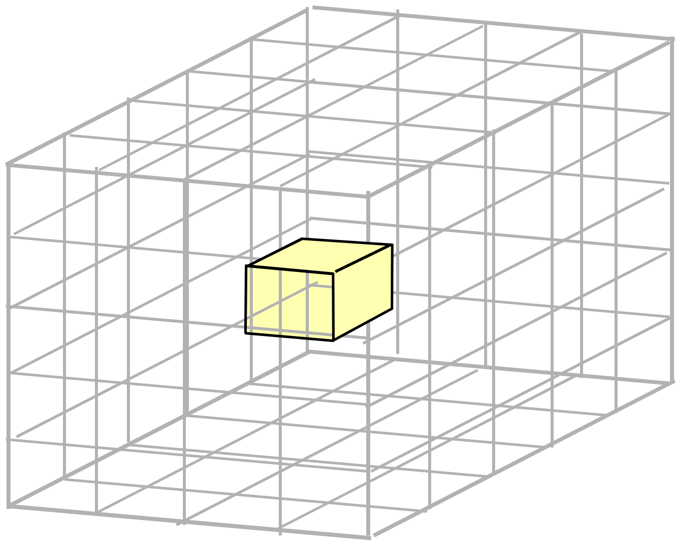






jorge s. marques, 2009

Histograma de cor



contar o número de pixels que pertence (vota) em cada célula.

h_k - número de pixels da célula k

A informação espacial é destruída no cálculo do histograma.

O espaço RGB é muito usado mas ou outros espaços podem também ser usados.

interpretação probabilística

Pode-se assumir-se que os pixels da imagem são realizações de uma variável aleatória discreta com distribuição

$$P(k) = C\, h_k \qquad \text{C – constante de normalização}$$

A comparação entre imagens passa a ser uma comparação entre distribuições de probabilidade.

comparação de histogramas

norma euclidiana $\sum_k [P_1(k) - P_2(k)]^2$

distância do mínimo $\sum_k \min\{P_1(k), P_2(k)\}$

divergência de Kullback $\sum_k P_1(k) \log \frac{P_2(k)}{P_1(k)}$